Taylor’s Guide to Scales

Taylor Hummon

Shift operators

Shift operators formalize a familiar musical idea: moving a perfect fifth. The *shift up operator* is drawn as an upward pointing arrow and is defined on natural notes as follows.

A🡑 = E
B🡑 = F♯
C🡑 = G
D🡑 = A
E🡑 = B
F🡑 = C
G🡑 = D

We'll extend our definition to non-natural notes by declaring that our operator can slide past sharp and flat symbols for free.

♯🡑 = 🡑♯
♭🡑 = 🡑♭

Let's start to get a feel for the shift up operator by looking at a few examples!

e.g. Let's simplify the expressions C♯🡑, A♭🡑, and B♭🡑.

C♯🡑 = C🡑♯ = G♯

A♭🡑 = A🡑♭ = E♭

B♭🡑 = B🡑♭ = F♯♭ = F

In the last case we simplified F♯♭ by canceling the sharp with the flat.

We can use the shift up operator multiple times in a row.

e.g. Let's look at using the shift up operator multiple times on the note, F.

F🡑 = C

F🡑🡑 = C🡑 = G

F🡑🡑🡑 = C🡑🡑 = G🡑 = D

And we can use exponents on the shift up arrow as a shorthand. In particular we could write

F🡑🡑🡑 as F$🡑^3$

to avoid repetition.

By repeatedly using the shift up operator, we produce an infinite ladder of notes! And by flipping all of the arrows upside down, we find the inverse, the *shift down operator*.

⋮	⋮
🡑	🡓
F♯	F♯
🡑	🡓
B	B
🡑	🡓
E	E
🡑	🡓
A	A
🡑	🡓
D	D
🡑	🡓
G	G
🡑	🡓
C	C
🡑	🡓
F	F
🡑	🡓
B♭	B♭
🡑	🡓
⋮	⋮

Let's try out the shift down operator with a few examples.

e.g. Let's simplify the expressions A🡓, B♭🡓, and C♯🡓.

A🡓 = D

B♭🡓 = B🡓♭ = E♭

C♯🡓🡓 = C🡓🡓♯ = F🡓♯ = B♭♯ = B

Just like how sharps and flats cancel each other, the shift up and shift down operators cancel each other.

e.g. We can use cancellations to rewrite equations.

B🡑 = F♯	
B🡑🡓 = F♯🡓	(shift down both sides of equation)
B = F🡓♯	(cancel shift up with shift down)
B♭ = F🡓♯♭	(flat both sides of equation)
B♭ = F🡓	(cancel sharp with flat)

Spelling scales

In this section we're going tackle spelling scales. But before we get there, we need a way to name scales. For that we'll turn to keys.

A *key* consists of two pieces of information: a number called the *degree* and a natural note called the *mode*. We'll write keys as follows.

```
key = degree.mode
```

Every key will give us a scale, and different keys will give us different scales.

> A *scale* is a list of notes that we build from a key as follows.
>
> 1. We list the seven natural notes in alphabetical order starting with the key's mode note.
> 2. We then shift up each of the notes in the list as many times as the degree tells us.
>
> We'll call the first note in the scale the *root note*.

In our first few examples, we'll stick to the mode of C which is by far and away the most popular mode. Whenever we use mode note C, we get *major scales*.

e.g. Let's start by spelling the scale given by the key, 0.C. Our first step is to write the seven natural notes in alphabetical order starting with C.

```
C    D    E    F    G    A    B
```

Our second step normally tells us to shift each of these notes. But our key degree is zero, so there's no shifting needed! In other words, we're already done spelling our first scale. Strictly speaking, we've spelled a seven note scale. If you'd prefer to have an eight note scale, just repeat the root note at the end of the list.

```
C    D    E    F    G    A    B    C
```

e.g. We'll next spell the degree-one scale, 1.C. We start just as before with the seven natural notes in alphabetical order beginning at mode note C.

```
C    D    E    F    G    A    B
```

Now we'll shift each of these notes up once.

```
C🡅   D🡅   E🡅   F🡅   G🡅   A🡅   B🡅
G    A    B    C    D    E    F♯
```

For an eight note scale, we'll repeat root note G.

```
G    A    B    C    D    E    F♯   G
```

e.g. Let’s spell the scale, `-1.C`. This time we shift notes down by one.

C↓	D↓	E↓	F↓	G↓	A↓	B↓
F	G	A	B♭	C	D	E

Nice!

We get *minor scales* whenever we use mode note A.

e.g. Let’s look at a few minor scales. First, let’s look at the degree-zero scale, `0.A`. There's no shifting to do, so spelling our scale is easy.

A	B	C	D	E	F	G

Now let's spell the degree-one scale, `1.A`. We’ll take each of the notes from our degree-zero scale and shift them up once.

A↑	B↑	C↑	D↑	E↑	F↑	G↑
E	F♯	G	A	B	C	D

And let’s also spell the scale, `-1.A`.

A↓	B↓	C↓	D↓	E↓	F↓	G↓
D	E	F	G	A	B♭	C

If scales share have the same degree, then they have the same set of notes!

e.g. Let’s look at three scales with degree two. We’ll start with the major scale, `2.C`.

C↑^{2}	D↑^{2}	E↑^{2}	F↑^{2}	G↑^{2}	A↑^{2}	B↑^{2}
D	E	F♯	G	A	B	C♯

Now here’s the minor scale, `2.A`.

A↑^{2}	B↑^{2}	C↑^{2}	D↑^{2}	E↑^{2}	F↑^{2}	G↑^{2}
B	C♯	D	E	F♯	G	A

And here's the scale, `2.F`.

F↑^{2}	G↑^{2}	A↑^{2}	B↑^{2}	C↑^{2}	D↑^{2}	E↑^{2}
G	A	B	C♯	D	E	F♯

Each of these degree-two scales start on different notes, but they all use the same set of notes.

Key signatures

Last section we saw how to spell scales using shift operators. In this section we're going to take a look at key signatures. A key signature gives us a second way to describe what notes are used in a scale, and it's often less work to find the key signature than it is to use shift operators.

> The *key signature* for a scale is a list of notes built as follows.
>
> - If the scale's degree is positive, say n, we start with notes
>
>   F, C, G, D, A, E, B
>
>   and upgrade n notes. We *upgrade* by crossing out the left-most note and then adding it back, sharped, on the right.
> - If the scale's degree is negative, say -n, we start with notes
>
>   B, E, A, D, G, C, F
>
>   and downgrade n notes. We *downgrade* by crossing out the left-most note and then adding it back, flatted, on the right.
>
> The key signature depends on a scale's degree and is independent of the mode.

e.g. Let's find the key signature for degree-three scales. So we'll start with

F, C, G, D, A, E, B

and upgrade three notes. Our left-most note name is F. We'll upgrade it by crossing it out and writing F♯ on the right side of our list.

~~F~~, C, G, D, A, E, B, F♯

Now our left-most note name is C. Let's upgrade C.

~~F~~, ~~C~~, G, D, A, E, B, F♯, C♯

And by upgrading G, we get our key signature.

~~F~~, ~~C~~, ~~G~~, D, A, E, B, F♯, C♯, G♯

All degree-three scales use this set of seven notes.

e.g. Let’s spell the scale, `-2.C`, by finding its key signature. We’re using a negative degree, so we’ll start with the following notes.

`B, E, A, D, G, C, F`

First we’ll downgrade B,

~~B~~, E, A, D, G, C, F, B♭

and then we'll downgrade E to give our key signature.

~~B~~, ~~E~~, A, D, G, C, F, B♭, E♭

We’ve found the seven notes of our scale! But they’re not yet in the correct order for a scale. To know what note comes first, we’ll need to find our scale’s root. Our key is `-2.C`, and so our root note is two shifts down from mode note C.

$$C\uparrow^{-2} = C\downarrow\downarrow = B\flat$$

OK, we now have everything we need to spell our scale. We begin on B♭, and we progress through the notes in the key signature alphabetically.

B♭ C D E♭ F G A

This is the scale, `-2.C`.

> The degree of a scale tells us
>
> - the number of sharps or flats in the key signature, or equivalently,
> - the number of sharps or flats in the seven note scale.
>
> We get sharps when the degree is positive and flat when the degree is negative.

Let’s see an example of a scale that uses a lot of flats. While there isn’t much practical need to use more than six sharps or flats, it’s fun to see that you can!

e.g. Let’s find the scale, `-10.A`. It would be a lot of effort to use ten shift operators on seven notes, so let’s use a key signature instead. Downgrading notes ten times gives us the following key signature.

~~B~~, ~~E~~, ~~A~~, ~~D~~, ~~G~~, ~~C~~, ~~F~~, ~~B♭~~, ~~E♭~~, ~~A♭~~, D♭, G♭, C♭, F♭, B♭♭, E♭♭, A♭♭

We can check our work by ensuring that there are ten flats in the key signature. OK, I see ten!

Now we need to find our scale's root note. We'll shift mode note A down ten times.

$$A\uparrow^{-10} = A\downarrow^{10} = C\flat$$

We're ready to spell our scale.

C♭ D♭ E♭♭ F♭ G♭ A♭♭ B♭♭

Notice how our seven note scale has ten flats. If we wrote our scale as an eight note scale,

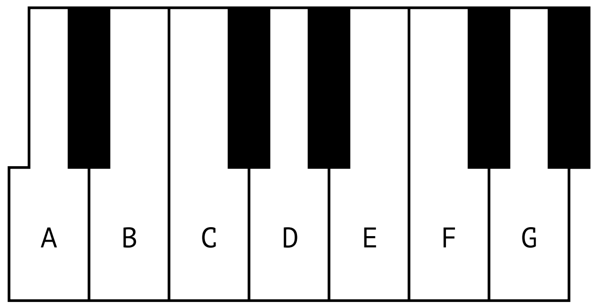
C♭ D♭ E♭♭ F♭ G♭ A♭♭ B♭♭ C♭

it would be easy to mistakenly think the scale was degree negative eleven.

Enharmonic equivalence and chromatic steps

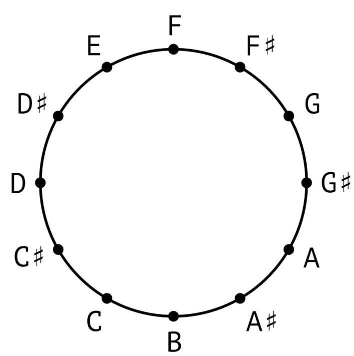
Up until this point we've been very precise with how we refer to notes. It's no good to say B♭ when you actually meant A♯ when you're spelling a scale. Moving forward we can be more relaxed.

An octave's worth of notes on a piano has seven white keys and five black keys. We've labeled the white keys using natural notes as usual.



We can move up or down chromatic steps using sharps and flat. While C♯♯ and D refer to the same white key on the keyboard, strictly speaking, these are unequal notes. Instead we'll say that C♯♯ and D are *enharmonically equivalent notes*. For most purposes—apart from spelling scales—we can treat these as the same note.

Let's draw all twelve notes of an octave spaced evenly around a circle. The result is the *circle of chromatic steps*.



I've chosen to write notes using sharps, but flats would work just fine too. I decided to start with F at the top of the circle, and each step clockwise corresponds to moving up the scale one chromatic step.

e.g. Let's count the number of chromatic steps going clockwise from C to E. We'll draw each step as a rounded arrow that looks like clockwise movement.

C ⤻ C♯ ⤻ D ⤻ D♯ ⤻ E

Despite involving five notes, there are only four arrows and so there are four steps!

Let's also count steps moving counter-clockwise from C to E. We'll draw counter-clockwise steps as arrows rounded the other way.

C ⤻ B ⤻ A♯ ⤻ A ⤻ G♯ ⤻ G ⤻ F♯ ⤻ F ⤻ E

I count eight counter-clockwise steps.

Here are two fun facts about the shift operators.

- The shift up operator 🠅 moves seven chromatic steps clockwise, and
- the shift down operator 🠇 moves seven chromatic steps counter-clockwise.

e.g. If we shift up once from F, we arrive at C.

F🠅 = C

There are seven chromatic steps moving clockwise from F to C.

F ⤼ F♯ ⤼ G ⤼ G♯ ⤼ A ⤼ A♯ ⤼ B ⤼ C

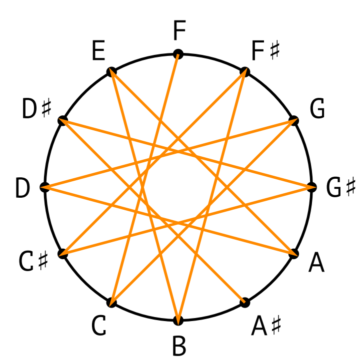
And if we shift up once from C, we arrive at G.

C🠅 = G

Once again, there are seven chromatic steps moving clockwise from C to G.

C ⤼ C♯ ⤼ D ⤼ D♯ ⤼ E ⤼ F ⤼ F♯ ⤼ G

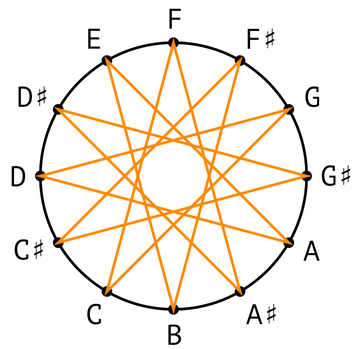
Let's draw a picture of repeatedly using the shift up operator. We'll draw each shift as a line segment within the circle of chromatic steps. Just as in the previous example, we'll start at F, then move to C, then G. Continuing in this way, we'll travel to all twelve chromatic notes in the octave. The twelfth note we travel to is A♯.



Shifting up once from A♯, we arrive at E♯.

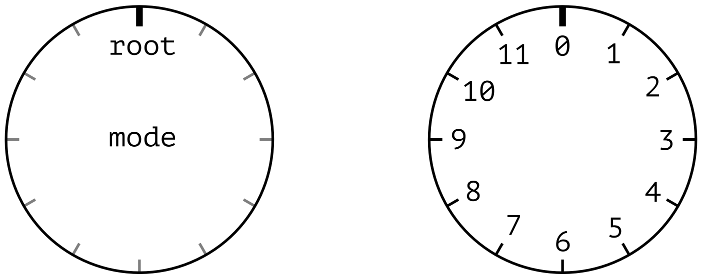
A♯↑ = A↑♯ = E♯

If we're being very strict about how we refer to notes, E♯ is not on our circle. But E♯ is enharmonically equivalent to F, which was the first note on our circle. Making this last connection, we've drawn a twelve pointed star!



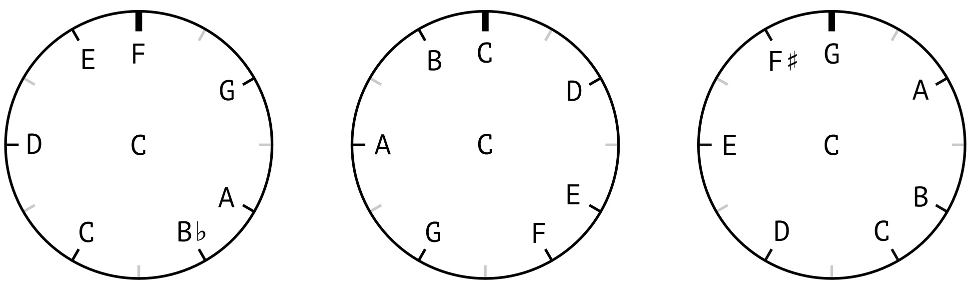
Scale charts

In this section we’re going to visualize scales with a technique called a *scale chart*. We start by drawing a circle with twelve markings that is reminiscent of an analog clock face. We write the mode note in the center of the circle and the root note at the top position. Each marking represents a chromatic note very much like in the circle of chromatic steps. We’ll draw the seven notes of the scale around the circle skipping markings where the scale skips notes.



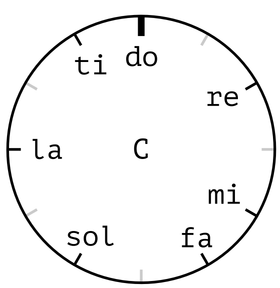
We can refer to the markings around the circle using the language of clocks, say, `3` o’clock or `10` o’clock. But we have one important exception—we’ll refer to the top marking as `0` o’clock instead of `12` o’clock.

Here are scale charts for the major scales with one flat, no sharps or flats, and one sharp.

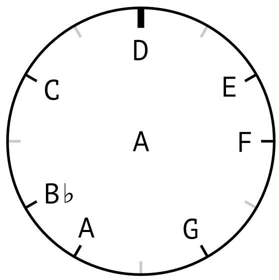


`-1.C` `0.C` `1.C`

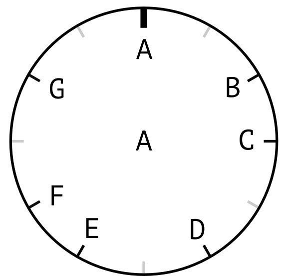
Each chart for a major scale has notes at the same positions: `0`, `2`, `4`, `5`, `7`, `9`, and `11` o’clock. We can write the *solfège notes* `do`, `re`, `mi`, etc. at these hour positions to produce the *chart for all major scales*.



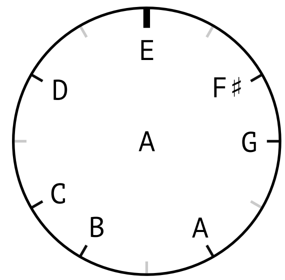
Here are the scale charts for minor scales that have one flat, no sharps or flats, and one sharp.



-1.A

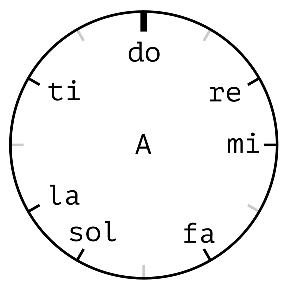


0.A

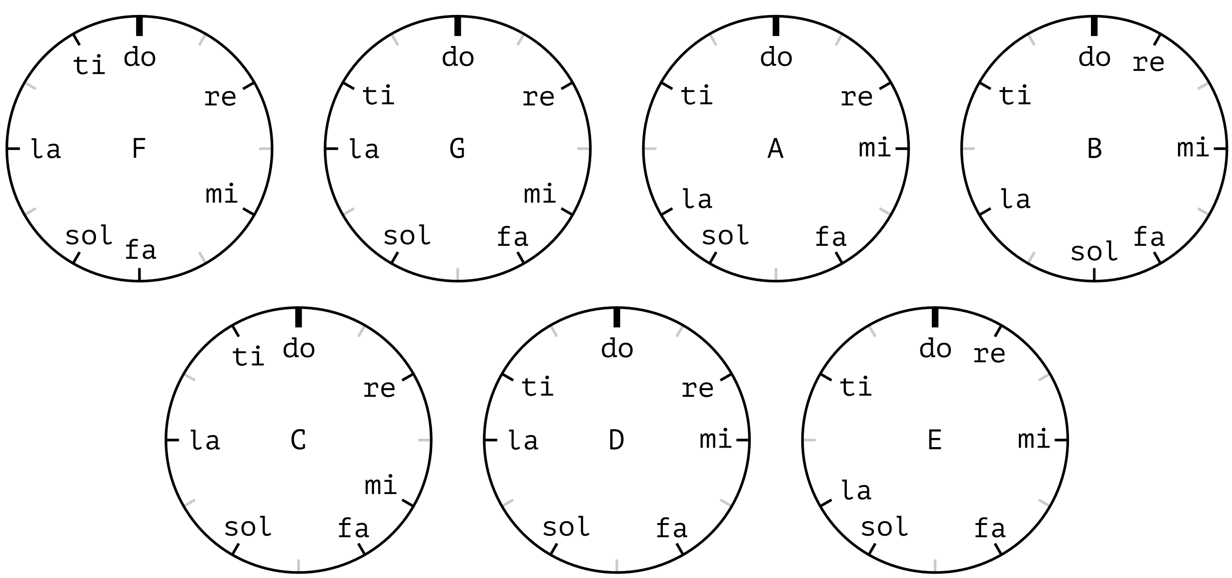


1.A

These minor scale charts all have notes at `0`, `2`, `3`, `5`, `7`, `8`, and `10` o'clock. These are a different set of positions than we saw for major scales. We can draw a *chart for all minor scales* again using solfège notes.



We've now seen a chart for major scales and a chart for minor scales. All seven modes have a distinct chart.



The following table has the same information as the charts above. For any mode and any solfège note, we can read where that note is positioned on its chart.

note	F	C	G	D	A	E	B	possibilities
do	0	0	0	0	0	0	0	0
re	2	2	2	2	2	1	1	1 or 2
mi	4	4	4	3	3	3	3	3 or 4
fa	6	5	5	5	5	5	5	5 or 6
sol	7	7	7	7	7	7	6	6 or 7
la	9	9	9	9	8	8	8	8 or 9
ti	11	11	10	10	10	10	10	10 or 11

Apart from do, each solfège note has two possible positions that are an hour apart: an early and a late position. For example, `re` can show up at 1 o'clock (early) or 2 o'clock (late). Working with mode note F gives all late positions, and working with mode note B gives all early positions. Each mode sits on a spectrum between these two extremes.

Standard triads

This section is a quick introduction to spelling and classifying the seven standard triads. Before we can spell a standard triad, we need to choose a scale to work in. Once we've made up our mind, we'll *spell standard triads* as follows.

- We start with a note in the scale which we'll call the *triad's root*.
- We take the note in the scale two above the triad's root.
- We take the note in the scale four above the triad's root.

Let's see how this looks with a few examples.

e.g. Let's work in the scale, `2.C`.

```
D     E     F♯    G     A     B     C♯
```

This scale has root note `D`. If we choose to also use `D` as our triad's root, we find the triad:

```
D, F♯, A.
```

We can build standard triads on any of the scale's seven notes. The triad built on `E` is

```
E, G, B.
```

The triad built on `F♯` is

```
F♯, A, C♯.
```

And so on.

Recall that the notes of a scale are determined by the scale's degree—the mode doesn't affect the set of notes used by a scale. As such, any standard triad in `2.C` is also a standard triad in `2.A` or any other degree-two scale.

e.g. Let's take a quick look at standard triads for the scale, `2.A`.

```
B     C♯    D     E     F♯    G     A
```

Each of the triads we built in the previous example are also standard triads for `2.A`.

```
D, F♯, A
E, G, B
F♯, A, C♯
```

And so on.

We’re going to give names to each of the seven standard triads. Here’s how to spell the standard triads in degree-zero scales.

name	root	notes
Aeolian triad	A	A, C, E
Locrian triad	B	B, D, F
Ionian triad	C	C, E, G
Dorian triad	D	D, F, A
Phrygian triad	E	E, G, B
Lydian triad	F	F, A, C
Mixolydian triad	G	G, B, D

For scales of other degrees, we’ll shift the notes of the standard triad appropriately.

e.g. For degree-zero scales, the Lydian triad is

F, A, C.

In degree one, the Lydian triad is

F🡑, A🡑, C🡑 = C, E, G.

And in degree two, the Lydian triad is

F🡑🡑, A🡑🡑, C🡑🡑 = G, B, D.

e.g. In degree zero, the Aeolian triad is

A, C, E.

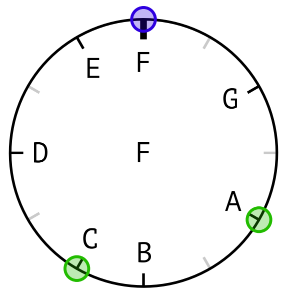
In degree one, the Aeolian triad is

A🡑, C🡑, E🡑 = E, G, B.

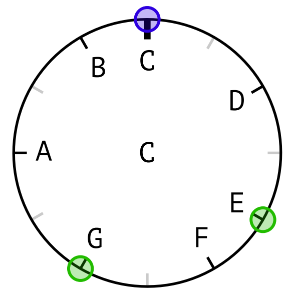
And in degree two, the Aeolian triad is

A🡑🡑, C🡑🡑, E🡑🡑 = B, D, F♯.

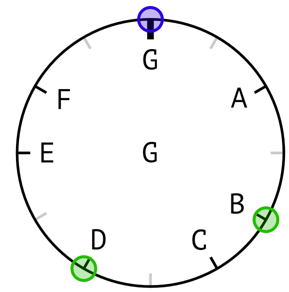
Let’s draw standard triads on scale charts so that we can take a look at their chromatic shapes. Here are the Lydian, Ionian, and Mixolydian triads. I’ve arranged the charts so that the triads’ roots are each at `0` o’clock.



Lydian



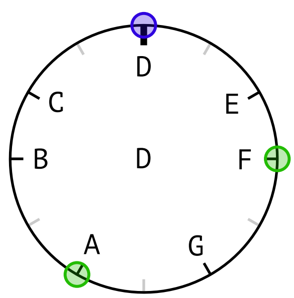
Ionian



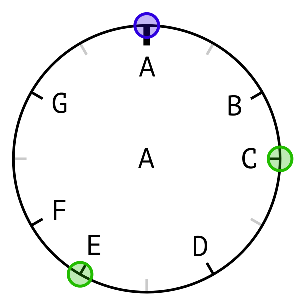
Mixolydian

We see that these triads have the same chromatic shape! From the root note, it’s four chromatic steps to the next note. Then it’s three chromatic steps. And then five chromatic steps gets us back to the root note. Whenever we see this shape, we’ll say that we have a *major triad*. In other words, the Lydian, Ionian, and Mixolydian triads are major.

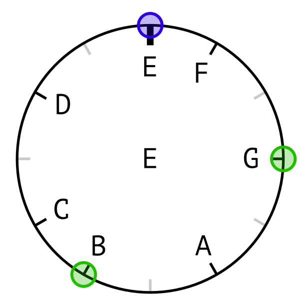
Similarly, the Dorian, Aeolian, and Phrygian triads have a common chromatic shape.



Dorian

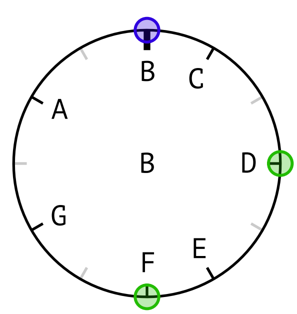


Aeolian



Phrygian

Starting at the root note, it's three chromatic steps, then four, and then five to return back to the root note. We’ll say that the Dorian, Aeolian, and Phrygian triads are *minor triads*. Last, let’s take a look at the Locrian triad.



Locrian

The Locrian triad has a pattern of chromatic shape all its own. From the root note, it’s three chromatic steps, then three, and six to return. We'll say that the Locrian triad is *diminished*.

e.g. Let's look at the seven standard triads for the major scale with three sharps, `3.C`. We'll start by figuring out how to convert between degree-zero notes and degree-three notes.

degree 0	**degree 3**
C	A
D	B
E	C♯
F	D
G	E
A	F♯
B	G♯

Now it's easy to spell the standard triads. For example, the Ionian triad is spelled in degree zero as

`C, E, G.`

So in degree three, the Ionian triad is

`A, C♯, E.`

Here are all seven standard triads for `3.C`.

chord	root	notes	quality
Ionian triad	A	A, C♯, E	major
Dorian triad	B	B, D, F♯	minor
Phrygian triad	C♯	C♯, E, G♯	minor
Lydian triad	D	D, F♯, A	major
Mixolydian triad	E	E, G♯, B	major
Aeolian triad	F♯	F♯, A, C♯	minor
Locrian triad	G♯	G♯, B, D	diminished

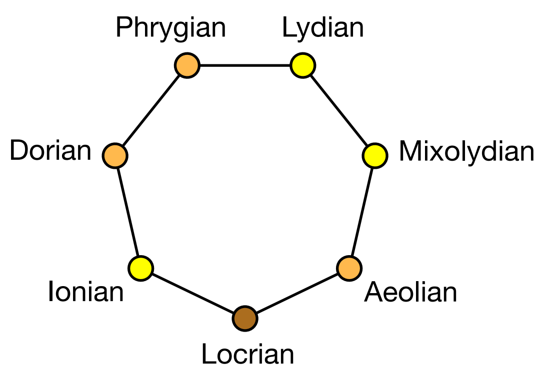
e.g. Let's now look at the standard triads for the minor scale with two flats, `-2.A`.

degree 0	**degree -2**
A	G
B	A
C	B♭
D	C
E	D
F	E♭
G	F

We've figured out how to translate between degree-zero notes and degree-negative-two notes. And so here are the seven standard triads for `-2.A`.

chord	root	notes	quality
Aeolian triad	G	G, B♭, D	minor
Locrian triad	A	A, C, E♭	diminished
Ionian triad	B♭	B♭, D, F	major
Dorian triad	C	C, E♭, G	minor
Phrygian triad	D	D, F, A	minor
Lydian triad	E♭	E♭, G, B♭	major
Mixolydian triad	F	F, A, C	major

Let's end our guide with a drawing of the seven standard triads. We'll write the triads at the corners of a polygon, and we'll color the corners yellow for major triads, orange for minor triads, and brown for the diminished triad.



We've placed the triads so that their root notes are in alphabetical order as we move clockwise. When working with major scales, the standard triads start at Ionian. And when working with minor scales, the standard triads start at Aeolian.